

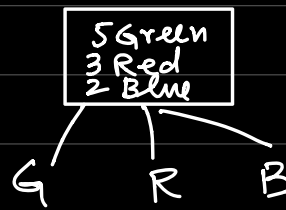
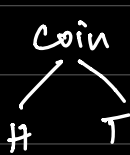
# BINOMIAL DISTRIBUTION

(WITHOUT REPLACEMENT CANNOT BE BINOMIAL) (P and q changes)

**CONDITIONS**

1. REPEATED EXPERIMENT ( $n$ )

2. DISCRETE (FIXED) OUTCOMES.



3. SUCCESS ( $p$ ) and FAILURE ( $q$ )

↓  
outcome being observed.

$$q = 1 - p$$

4.  $n$ ,  $p$  &  $q$  must be constants.

$$P(X=r) = {}^n C_r p^r q^{n-r}$$

outcome being observed (success) →  $r$   
desired value of success →  $n$

$$P1$$
$${}^n C_r a^{n-r} b^r$$

Be careful.

$n$  = no. of repeats

$p$  = probability of success

$q$  = probability of failure.

Q. A Fair dice is thrown 20 times.

X denotes the random variable for number of times that dice lands on a multiple of 3. Find the probability that:

$$n=20, p = \frac{2}{6} = \frac{1}{3}, q = \frac{2}{3}$$

(a) Dice lands on multiple of 3 exactly twice.  
Success  $X=2$

$$P(X=2) = {}^{20}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{18} = \boxed{\phantom{000000}}$$

means that when we throw the dice 20 times, we get success (multiple of 3) only 2 times and we get failure (1,2,4,5) for 18 times.

(b) Dice lands on multiple of 3 at least 18 times.  
Success  $X \geq 18$

$$\begin{aligned} P(X \geq 18) &= P(X=18) + P(X=19) + P(X=20) \\ &= {}^{20}C_{18} \left(\frac{1}{3}\right)^{18} \left(\frac{2}{3}\right)^2 + {}^{20}C_{19} \left(\frac{1}{3}\right)^{19} \left(\frac{2}{3}\right)^1 + {}^{20}C_{20} \left(\frac{1}{3}\right)^{20} \left(\frac{2}{3}\right)^0 \\ &= \boxed{\phantom{000000}} \end{aligned}$$

(c) Dice lands on multiple of 3 at most twice.  
Success  $X \leq 2$

$$\begin{aligned} P(X \leq 2) &= P(X=2) + P(X=1) + P(X=0) \\ &= {}^{20}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{18} + {}^{20}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{19} + {}^{20}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20} \\ &= \boxed{\phantom{000000}} \end{aligned}$$

(d) Dice lands on multiple of 3 at least once.

Success

$X \geq 1$

$$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) + \dots + P(X=20)$$

X
0
1
2
3
⋮
20

→ X = at least once

$$\begin{aligned}
 P(X \geq 1) &= 1 - P(X=0) \\
 &= 1 - {}^{20}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20} \\
 &= \boxed{\phantom{000000}}
 \end{aligned}$$

(e) Dice lands on a multiple of 3 at least twice.

Success

$X \geq 2$

$$P(X \geq 2) = P(X=2) + P(X=3) + \dots + P(X=20)$$

X
0
1
2
3
⋮
20

X = at least twice

$$\begin{aligned}
 P(X \geq 2) &= 1 - P(X=0) - P(X=1) \\
 &= 1 - {}^{20}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20} - {}^{20}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{19} \\
 &= \boxed{\phantom{000000}}
 \end{aligned}$$